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Students' Conceptual and Procedural Knowledge on Integration: Reflections on Calculus Learning

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Abstract. This study aims to describe conceptual and procedural knowledge on integral topics. Data were collected from 30 students who were randomly selected from 120 students of the mathematics education study program at IAIN Bukittinggi who took an integral calculus course. Data were analyzed by categories from student's work. The results showed that students' conceptual knowledge is still very low. Students do not understand the meaning of the definite integral as the limit of the Riemann sum and build a definition of the integral. Most of the students had a fairly good knowledge of procedural aspects. Students can determine the integral of a function using the basic theorems of calculus and can use the right integration technique for some given functions. On the other hand. Integral calculus learning is expected to increase students' conceptual knowledge in addition to their procedural knowledge.

INTRODUCTION

Calculus is one of the topics in universities for the Mathematics Education study program. According to Mahir [1], Calculus is an important and fundamental subject for mathematics students. Calculus comes from the word "calculi" which means "calculation system" [2]. Calculus is a branch of mathematics that includes differential calculus and integral calculus [3]. Several studies reveal the difficulties experienced by students related to the concept of Calculus, including research conducted by many experts. This also happened in the Mathematics Education study program at IAIN Bukittinggi. Most students' Calculus scores are below average due to a lack of understanding of the concept of Calculus [4].

The main principle of understanding is the capacity to make connections between conceptual knowledge and procedural knowledge [5]. Someone will have a strong understanding of mathematics when they can connect the two pieces of knowledge. These two terms have been commonly used in mathematics learning since the mid-1980s. This framework is the basis for setting goals, how to achieve and evaluate these learning goals [6]. Both of this knowledge is important to develop properly [7-8]. Lack of conceptual knowledge hinders students' capacity to transfer and generalize [9]. Spending more time on conceptual is more beneficial than time spent teaching a procedure when the goal is for a stronger understanding of concepts and procedures [10].

Conceptual knowledge is rich in relationships in the form of connected knowledge networks [11], more complex and organized [5]. More specifically Johnson [5] and Vanchoy [12] define conceptual knowledge as an explicit and implicit understanding of definitions, rules, and principles as well as the interrelationships between parts of knowledge in a particular domain. Procedural knowledge is knowledge of how to do something, including knowledge of algorithms, techniques, and methods, as well as knowledge of specific criteria or strategies used to solve problems [5, 11-13]. Serhan [14] divides procedural knowledge into two different parts, namely formal language, or symbol representation systems, mathematics, and algorithms, or rules.

One of the causes of the lack of understanding of mathematical concepts, especially calculus, is learning that does not develop procedural knowledge as well as conceptual knowledge. This is following the opinion of Doorman [15] which says that students' understanding of the concept of Calculus is low because Calculus students emphasize more on routine things and focus on how to derive or integrate a function and rarely discuss concepts. This is also reinforced

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by research conducted by Serhan [14] which reveals that students' procedural knowledge in Integral topics is certainly more dominant than conceptual knowledge. Students can determine definite integrals but cannot explain the concept of definite integrals. In addition to the concept of definite integral, another important concept in Calculus is the indefinite integral. Based on the results of previous research, the authors are interested in seeing the conceptual and procedural knowledge of students of the Mathematics Education Study Program at IAIN Bukittinggi on the topic of definite and indeterminate integrals.

METHOD AND DATA SOURCE

This is descriptive research. The aim is to investigate students' conceptual and procedural knowledge on the topic integrals. Descriptive research according to Isaac and Michel is the research that aims to systematically describe the facts and the characteristics of a given population or area of interest.

The data sources used in this study were 30 students of the Mathematics Education Study Program at IAIN Bukittinggi. Students were randomly selected from 120 people who took the Integral Calculus course. Data were collected by giving a test consisting of 2 questions that tested conceptual knowledge and 2 questions that tested procedural knowledge on the Integral topic, namely:

1. Find two antiderivatives of the function $f(u) = \frac{1}{u} + 2$!
2. Determine $\int x \sin 2x \, dx$!
3. Explain the meaning $\int_a^b f(x)dx$!
4. Determine $\int_1^3 (-2x + 1) \, dx$!

Student answer sheets were analyzed with five categories, namely: students can answer with the true concepts/ procedures, true concepts/ procedures but there are few mistakes, true concepts/ procedures but there are many mistakes, and the concepts/ procedures used are false and there is no answer.

RESULT AND DISCUSSION

Conceptual Knowledge

There are two questions that are tested for conceptual knowledge students on integral. The questions about concepts indefinite and definite integral:

1. Find two antiderivatives of the function $f(u) = \frac{1}{u} + 2$!

   Students have not been able to understand the concept that anti-derivative is an indefinite integral of a given function. Many students make the mistake, no one used true concepts. The distribution of student answers can be seen in table 1 Distribution of student's conceptual knowledge on indefinite integral.

Table 1. Distribution Student's Conceptual Knowledge on Indefinite Integral

Categories	Students	
	Number of students	Percentage
True concepts	0	0
True concepts but there are few mistakes	5	16.67
True concepts but there are many mistakes	9	30
False concepts	14	46.67
No answers	2	6.67

Many students make the mistake of using derivative concepts to look for anti-derivative. However, the two concepts are contradictory to each other. This mistake can be seen in Figure 1.

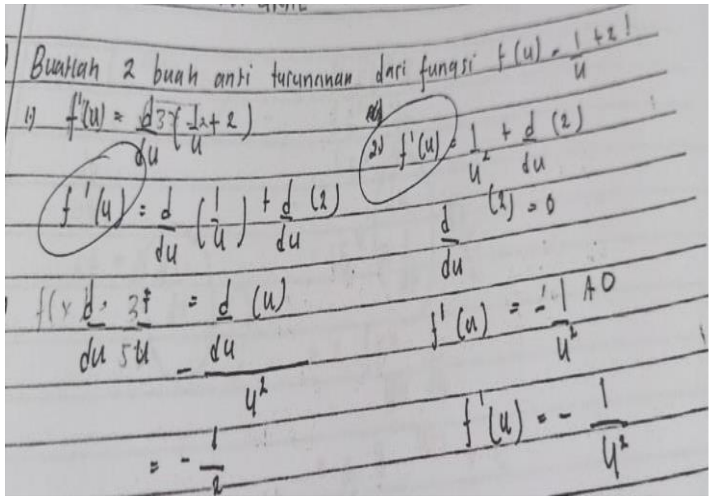

Figure 1. The Student Using Derivative Concepts to Look for Anti-derivative

Only a few of the students use the concept of Integral to determine the anti-derivative of a function, but students have not been able to understand the meaning of the constant on an indefinite integral so they cannot give two examples of the anti-derivative of the function. Conceptually, anti-derivatives of a function are infinite so that it is represented by addition with a constant c. The value of c can be replaced by any real number. This mistake can be seen in Figure 2.

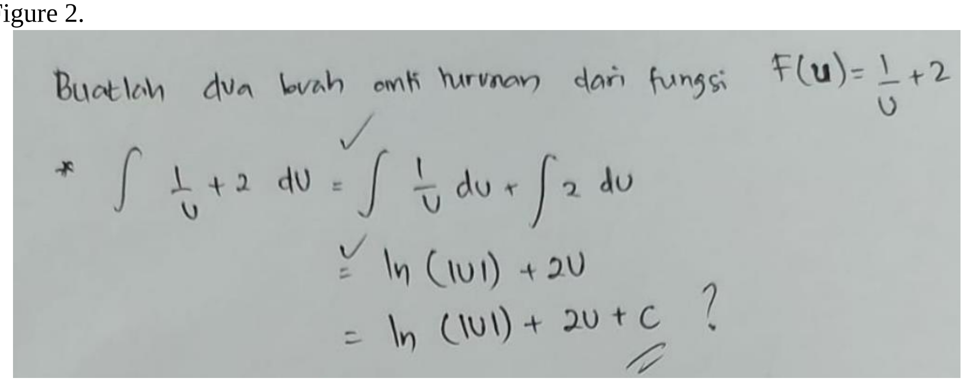

Figure 2. The Student Can't Determine Two Anti-derivatives a Function

2. Explain the meaning of $\int_a^b f(x)dx$!.

Students have not been able to understand the meaning of the definite integral. No one understands the definition of definite integral as a limit Riemann Sum. The distribution of student answers can be seen in table 2 Distribution of student's conceptual knowledge on definite integral.

Table 2. Distribution Student's Conceptual Knowledge on Definite Integral

Categories	Students	
	Number of students	**Percentage**
True concepts	0	0
True concepts but there are few mistakes	7	23.33
True concepts but there are many mistakes	20	66.67
False concepts	2	6.67
No answers	1	3.33

Many students just explain the notation of the definite integral of the given function and don't understand the meaning behind the symbol. This answer can be seen in Figure 3.

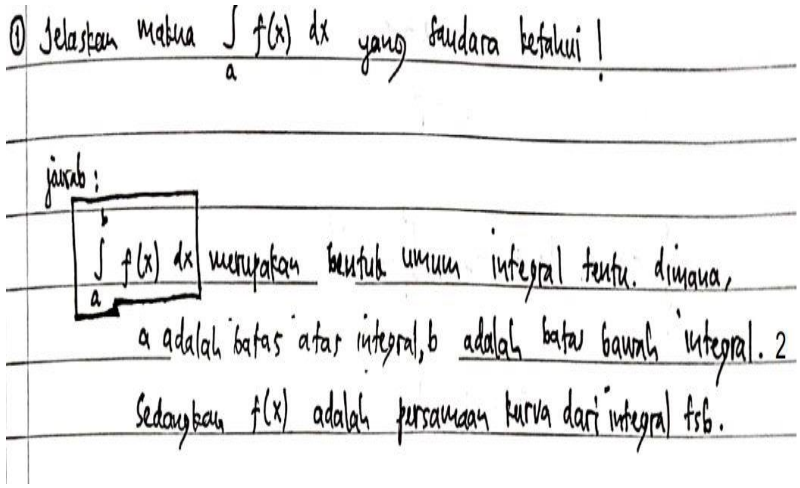

Figure 3. The Students Explain the Notation of the Definite Integral

Only a few students interpret the definite integral as the area below the curve that is limited by the lower boundary *a* and the upper limit *b*. This can be seen in Figure 4.

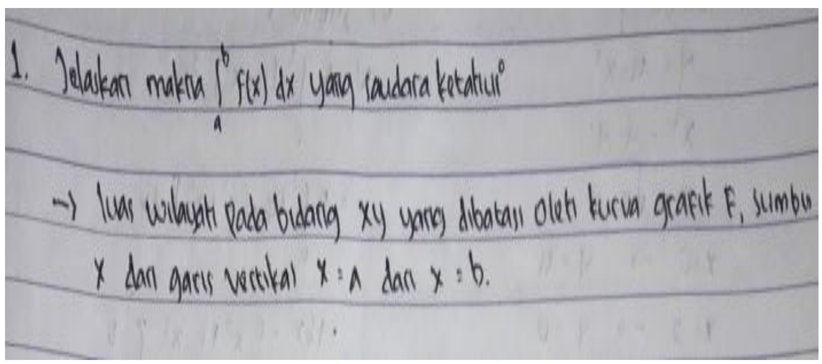

Figure 4. The Student Can't Determine Two Anti-derivatives a Function

Procedural Knowledge

There are two questions that are tested for procedural knowledge students on integral. The questions about determine indefinite and definite integral:

1. Determine $\int x \sin 2x \, dx$!

   Almost all students can determine the indefinite integral with the true procedure. Only a few students make a mistake. The distribution of student answers can be seen in Table 3

Table 3. Distribution Student's Procedural Knowledge on Indefinite Integral

Categories	Students	
	Number of students	Percentage
True procedures	23	76.67
True procedures but there are few mistakes	5	16.67
True procedures but there are many mistakes	0	0
False procedures	1	3.33
No answers	1	3.33

Students use the partial integral method by e.g., variable *x* with u and sin *x dx* as v'. This answer can be seen in Figure 5.

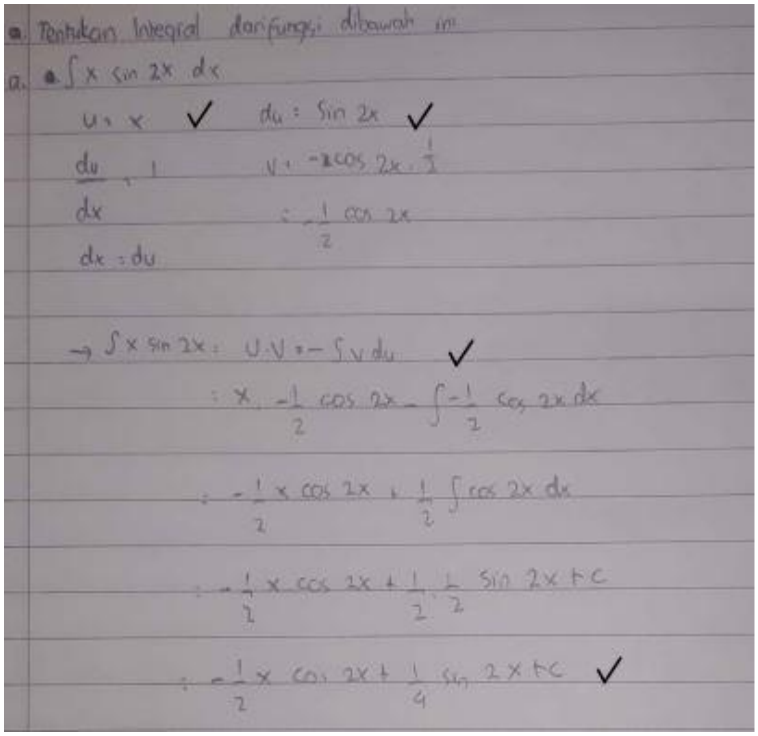

Figure 5. The Students Can Determine the Definite Integral of the Function

Only several students make a few mistakes when determining the indefinite integral of trigonometry functions. This can be seen in Figure 6.

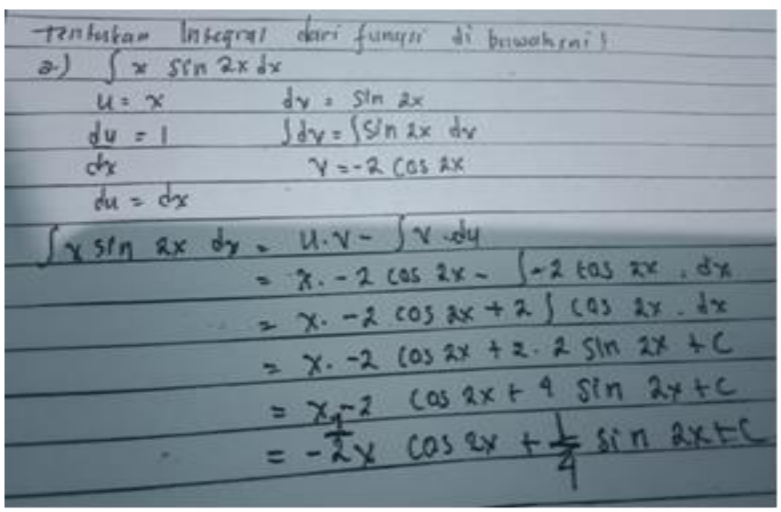

Figure 6. The Student Can't Determine Two Anti-derivatives a Function

2. Determine $\int_1^3 (-2x+1)\,dx$!

In general, the students can determine the definite integral with the true procedure. Many students used the calculus fundamental theorem but several of them make a few mistakes. The distribution of student answers can be seen in table 4

Table 4. Distribution Student's Procedural Knowledge on Definite Integral

Categories	Students	
	Number of students	**Percentage**
True procedures	9	30
True procedures but there are few mistakes	9	30
True procedures but there are many mistakes	3	10
False procedures	2	6.67
No answers	7	23.33

Many students used the calculus fundamental theorem but several of them make a few mistakes. These answers can be seen in Figure 7 and Figure 8**.**

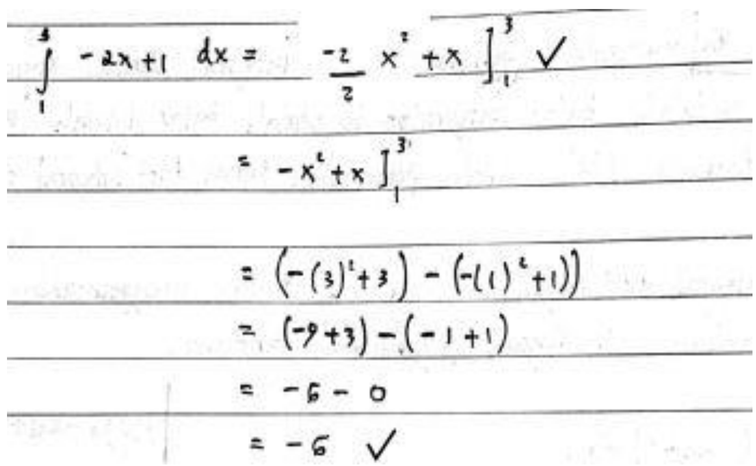

Figure 7. The Students Can Determine the Definite Integral of The Function

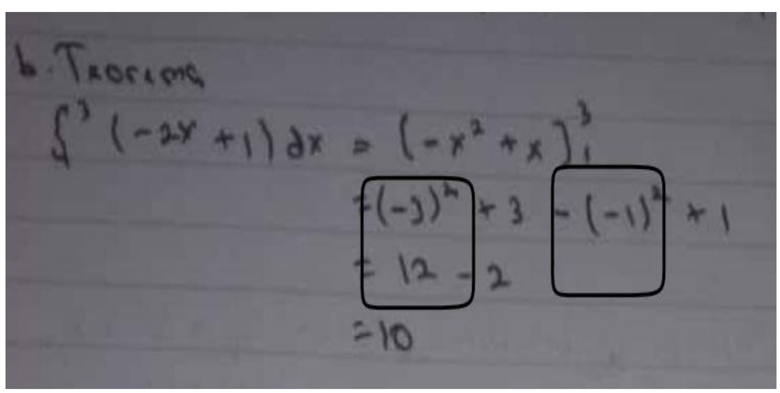

Figure 8. The Students Make a Few Mistake

The results of this study reveal that most students have not been able to solve problems that test conceptual knowledge. Students have difficulty in understanding the definition of integral, both definite integral and indefinite integral. This is in line with research conducted by Grundmeier TA, Hansen J, and Sousa [16]; Dormann [15]; D Hidayat, A W Kohar, & R Artiono [17]. On the other hand, students' procedural knowledge seems to be quite good. Most students have been able to solve problems using the correct procedure.

Therefore, students' procedural knowledge is better than conceptual knowledge on integrals. This is following the opinion of Edwards [18]; Kaput [19]; Doormann [15] which says that one can mechanically use the procedure of integral notation without thinking about its meaning. Research by Doormann [15] and Radmehr F & Drake M [20] reveals that this happens because in learning educators prioritize procedural knowledge rather than conceptual knowledge. According to Grundmeier [16] calculus, teachers should prioritize conceptual knowledge over procedural knowledge. This is so that calculus learning is not trapped in routine calculations. This is where the expertise of a teacher is required in designing and managing to learn. So that learning becomes more meaningful and student understanding becomes more comprehensive.

CONCLUSION

Based on the results of the study, it can be concluded that students' conceptual knowledge of the Integral is very low. The student can't understand that the antiderivative is an indefinite integral of a function and the number of antiderivatives is infinite. Likewise with the definite integral concept. Students have quite shallow conceptual knowledge. Most students only read symbols or notations of definite integrals. None of the students understood the definite integral as the Riemann sum of the functions partitioned with the partition distance to zero. On the other hand, the procedural knowledge of students of the Mathematics Education Study Program at IAIN Bukittinggi on the topic of indefinite integrals is better. Students can determine the integral of a given function using the true procedure. Likewise with determining the definite integral of a function using the basic theorem of calculus.

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